SF2722 Differential Geometry

Course at advanced level (course number KTH: SF2722, SU: MM8022), 7.5 credits, spring 2021.

ZOOM room for lectures: https://kth-se.zoom.us/j/67839417630 (https://kth-se.zoom.us/j/67839417630)
ZOOM room for exercise sessions: https://kth-se.zoom.us/j/69019331136 (https://kth-se.zoom.us/j/69019331136)

Discussion forum: We encourage everyone to participate actively in the **discussion forum (https://canvas.kth.se/courses/21942/discussion_topics/153968)** .

Teachers: **Mattias Dahl (https://www.kth.se/profile/dahl/)** and **Hans Ringström (https://people.kth.se/~hansr/)** .

Exercise sessions: **Bernardo Fernandes (https://www.kth.se/profile/bfer)**

Administration: Contact the **student affairs office (https://www.kth.se/sci/kontakt/studentexpedition/matematik/studentexpedition-matematik-1.35739)** for registration and other administrative matters.

Schedule: see **here (https://cloud.timeedit.net/kth/web/public01/ri15Q60Yg05057Q0g6QY7065Z169X393763Y510yZ05Q.html)** .

Literature:

- Christian Bär, lecture notes ***Differential geometry*** **(https://www.math.uni-potsdam.de/fileadmin/user_upload/Prof-Geometrie/Dokumente/Lehre/Lehrmaterialien/skript-DiffGeo-engl.pdf)** .
- John M. Lee, ***Introduction to smooth manifolds*** **(https://www.springer.com/gp/book/9781441999818)** (**KTH library link (https://link-springer-com.focus.lib.kth.se/book/10.1007%2F978-1-4419-9982-5)**).
- Peter Petersen, **Riemannian Geometry (https://www.springer.com/gp/book/9783319266527)** (**KTH library link (https://link-springer-com.focus.lib.kth.se/book/10.1007%2F978-3-319-26654-1)**).
- Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine, **Riemannian Geometry (https://www.springer.com/gp/book/9783540204930)** (**KTH library link (https://link-springer-com.focus.lib.kth.se/book/10.1007%2F978-3-642-18855-8)**).

As additional reading we recommend

- John M. Lee, *Introduction to topological manifolds*;
- John M. Lee, *Riemannian manifolds: an introduction to curvature*;

- Jeffrey M. Lee, *Manifolds and differential geometry*;
- William M. Boothby, *An introduction to differentiable manifolds and riemannian geometry*;
- Barrett O'Neill, *Semi-riemannian geometry*;
- Ben Andrews, ***Lectures on differential geometry*** (https://maths-people.anu.edu.au/~andrews/DG/);
- Christian Bär, lecture notes ***Theory of relativity*** (https://www.math.uni-potsdam.de/fileadmin/user_upload/Prof-Geometrie/Dokumente/Lehre/Lehrmaterialien/skript-Relativity-engl.pdf).

Description: The central objects in modern differential geometry are differentiable manifolds. In this course we will study differentiable manifolds and see how they are used to define concepts from analysis in a coordinate-independent way. In an introduction to (semi-)Riemannian geometry we will see how curvature is described. Ideas and methods from differential geometry are fundamental in modern physical theories.

Prerequisites: SF1677 **Foundations of Analysis** (https://www.kth.se/student/kurser/kurs/SF1677?l=en), or corresponding background. Also recommended is SF1678 **Groups and Rings** (https://www.kth.se/student/kurser/kurs/SF1678?l=en), or equivalent. A good knowledge of calculus of several variables including the inverse and implicit function theorems is an important prerequisite.

Content: The course is divided into seven modules with the following content:

- *Module 1: Manifolds.* This module covers the basic notions of a manifold, tangent vectors and vector fields. It corresponds to chapter 1 in Bär's notes.
- *Module 2: Semi-Riemannian Geometry.* In this module, the notion of a Semi-Riemannian metric is introduced, as well as the notion of parallel transport and geodesics. It corresponds to chapter 2 in Bär's notes.
- *Module 3: Curvature.* This module covers several notions of curvature: the Riemann curvature tensor, sectional curvature, Ricci curvature and scalar curvature. Jacobi fields are also discussed. It corresponds to chapter 3 in Bär's notes.
- *Module 4: Submanifolds.* In this module, we discuss how the geometry induced on a submanifold relates to the geometry of the ambient space. It corresponds to chapter 4 in Bär's notes.
- *Module 5: Flows and Lie brackets.* This module covers integral curves and flows of vector fields, as well as the notion of a Lie bracket and a simple version of Frobenius theorem. It corresponds to a part of chapters 8, 9 and 19 in Lee's book. We will also give an introduction to Killing vector fields, following chapter 8 in the book by Petersen.
- *Module 6: Riemannian geometry.* In this module, we give examples of how local geometric conditions can influence the global topology of a manifold. It corresponds to chapter 5 of Bär's notes.
- *Module 7: Continuation of Riemannian geometry. Differential forms, Stokes' theorem and de Rham cohomology.* In this module we first continue the study of Riemannian geometry, following parts of chapter 3 in the book by Gallot-Hulin-Lafontaine. After that we look at

differential forms, integration thereof, Stokes' theorem and de Rham cohomology. This corresponds to parts of chapters 14-18 in Lee's book.

Comments concerning the lectures. Comments concerning the lectures and exercise sessions can be found here (https://canvas.kth.se/courses/21942/pages/comments-on-the-lectures) .

Exercise sheets:

- Exercise sheet 0: Background on topology (https://canvas.kth.se/courses/21942/files/3749960/download?verifier=kHow160exd8a6hB5lvcneTYcGjsE6pF6Za3r5efx&wrap=1) (https://canvas.kth.se/courses/21942/files/3749960/download?verifier=kHow160exd8a6hB5lvcneTYcGjsE6pF6Za3r5efx&download_frd=1)
- Exercise sheet 1: Manifolds (https://canvas.kth.se/courses/21942/files/3749982/download?verifier=mCxadGPviYHMmavpd06hlQawdgdwlNHCBjOR5hzD&wrap=1) (https://canvas.kth.se/courses/21942/files/3749982/download?verifier=mCxadGPviYHMmavpd06hlQawdgdwlNHCBjOR5hzD&download_frd=1)
- Exercise sheet 2: Semi-Riemannian Geometry (https://canvas.kth.se/courses/21942/files/3750000/download?verifier=tCp5P5DJATQ3lRux43hPAaKdgi47VhnQ8Mw1RaaB&wrap=1) (https://canvas.kth.se/courses/21942/files/3750000/download?verifier=tCp5P5DJATQ3lRux43hPAaKdgi47VhnQ8Mw1RaaB&download_frd=1)
- Exercise sheet 3: Curvature (https://canvas.kth.se/courses/21942/files/3750038/download?verifier=82nQS9Pf51KvZDXCU49eoYSLjM0uTSeNLBzcuAZ7&wrap=1) (https://canvas.kth.se/courses/21942/files/3750038/download?verifier=82nQS9Pf51KvZDXCU49eoYSLjM0uTSeNLBzcuAZ7&download_frd=1)
- Exercise sheet 4: Submanifolds (https://canvas.kth.se/courses/21942/files/3750042/download?verifier=8mlQl2Bh8DUvFi9zfu67yyUhTb39wBz7UNq4loU9&wrap=1) (https://canvas.kth.se/courses/21942/files/3750042/download?verifier=8mlQl2Bh8DUvFi9zfu67yyUhTb39wBz7UNq4loU9&download_frd=1)
- Exercise sheet 5: Flows, Lie brackets, Killing fields (https://canvas.kth.se/courses/21942/files/4060695?verifier=3ff2xLyQXWDGdoC51GU68RSpJPJmawVu8Z9KQdLo&wrap=1) (https://canvas.kth.se/courses/21942/files/4060695/download?verifier=3ff2xLyQXWDGdoC51GU68RSpJPJmawVu8Z9KQdLo&download_frd=1)
- Exercise sheet 6: Riemannian geometry (https://canvas.kth.se/courses/21942/files/4055808?verifier=N3jPH2U5aJ7wolMxRCXK3X6N9OQdauGcimw3Bi5y&wrap=1) (https://canvas.kth.se/courses/21942/files/4055808/download?verifier=N3jPH2U5aJ7wolMxRCXK3X6N9OQdauGcimw3Bi5y&download_frd=1)
- Exercise sheet 7: Riemannian geometry. Differential forms, Stokes' theorem and de Rham cohomology (https://canvas.kth.se/courses/21942/files/4159937?verifier=KA749UEoFFbU1LQmfnfvwm2LejyhMC623ciHTri0&wrap=1) (https://canvas.kth.se/courses/21942/files/4159937/download?verifier=KA749UEoFFbU1LQmfnfvwm2LejyhMC623ciHTri0&download_frd=1)

Examination: The examination will be in the form of homework problems followed by an oral exam. For grades D-E you only have to solve the homework problems. For grades A-C you must also do the oral exam.

Homework problems: There will be 7 sets of homework problems, one for each module. They are to be found under **Assignments (https://canvas.kth.se/courses/21942/assignments)** . The problems will all be chosen from the exercise sheets.

We will not accept solutions handed in after the strict deadlines. Extra assignments to compensate any missed homework will be given at the end of the course.

Your solutions are handed in via **Assignments (https://canvas.kth.se/courses/21942/assignments)** . Make sure that scanned documents are in easily readable pdf format.

Oral exam: The oral exam takes place after all homework problems have been handed in and graded. In the oral exam you will be asked about problems from the exercise sheets, as well as theory and proofs from Christian Bär's lecture notes and John Lee's book. The oral exam will take place first week of June.