Kursanalys - KTH

Formulär för kursansvarig.
Kursanalysen utförs under kursens gång.
Nomenklatur: F – föreläsning, Ö – övning, R – räknestuga, L – laboration, S – seminarium)

KURSDATA Obligatorisk del

Kursens namn

Relativistisk kvantfysik

Kursnummer

SI2390

Kurspoäng och poäng fördelat på exam-former

7,5 hp (INL1, 4,5 hp och TEN1, 3 hp)

När kursen genomfördes

Läsåret 2023/2024 (period 3)

Kursansvarig och övriga lärare

Professor Tommy Ohlsson
Professor Sandhya Choubey

Undervisningstimmar, fördelat på F, Ö, R, L, S

6 x 2h F
12 x 2h F

Antal registrerade studenter 25

Prestationsgrad efter 1:a examenstillfället, i % 95,2

Examinationsgrad efter 1:a examenstillfället, i % 88,0

MÅL

Ange övergripande målen för kursen

Efter fullgjord kurs skall du kunna:
- tillämpa Poincarégruppen samt klassificera partikelrepresentationer.
- analysera Klein–Gordon- och Diracekvationerna.
- lösa Weylekvationen.
- känna till Maxwells ekvationer och klassisk Yang–Mills-teori.
- kvantisera Klein–Gordon-, Dirac- och Majoranafält samt ställa upp Lagrangetätheter för dessa fält.
- använda störningsteori inom enkla kvantfältteorier.
- ställa upp Lagrangetätheten för kvantelektrodynamik samt analysera denna.
- härleda Feynmanregler utifrån enkla kvantfältteorier samt tolka Feynmandiagram.
- analysera elementära processer i kvantelektrodynamik.
- beräkna strålningskorrektioner för elementära processer i kvantelektrodynamik.

Ange hur kursen är utformad för att uppfylla målen

Kursen är utformad så att föreläsningar och egna självstudier ska leda till att studenterna kan lösa skriftliga inlämningsuppgifter samt svara på teorifrågor och därmed uppfylla målen för kursen.

Eventuellt deltagande i länkmöte före kursstart

Synpunkter från detta

-

Kursens pedagogiska utveckling I

Beskriv de förändringar som gjorts sedan förra kursomgången. (Berätta även för studenterna vid kursstart)

-

Kontakt med studenterna under kursens gång

Studenter i årets kurs-nämnd:	**Namn**	**E-post** (lämnas blank vid webbpublicering)

Resultat av formativ mittkursenkät

Resultat av kursmöten

Kontakt med övriga lärare under kursens gång

Kommentarer

-

Kursenkät; teknologernas synpunkter Obligatorisk del

Att komma ihåg:
1) Uppmana, mha kursnämnden, till ifyllande av kursenkät i anslutning till / just efter slutexaminationen
2) Delge kursnämnden enkäten
3) Publicera enkäten under en kortare tid

Period, då enkäten var aktiv	2024-03-13 – 2024-03-29
Frågor, som adderades till standardfrågorna	• What is your overall impression of the course? • How would you rate the difficulty of the course? • Has there been much overlap with other courses? • How were the quizzes? • How were the (hand-in) homework problems? • How was the oral examination? • What is your opinion about the course description and the administration of the course? • What is your opinion about the course literature? • How were the lectures? (Sandhya Choubey) • How were the lectures? (Tommy Ohlsson) • Please enter any further comments on the course below.
Svarsfrekvens	52 %
Förändringar sedan förra genomförandet	-
Helhetsintryck	Enligt kursenkäten svarade majoriteten av studenterna att de var mycket eller ganska nöjda med kursen i sin helhet.
Relevanta webb-länkar	-

Kursansvarigs tolkning av enkät

Positiva synpunkter	Se bilaga.
Negativa synpunkter	Se bilaga.
Var kursen relevant i förhållande till kursmålen?	-
Syn på förkunskaperna	-
Syn på undervisningsformen	Föreläsningarna ansågs vara bra eller medelbra av en majoritet av studenterna.
Syn på kurslitt/kursmaterial	Kurslitteraturen ansågs vara bra.
Syn på examinationen	Inlämningsuppgifterna ansågs vara svåra av en majoritet av studenterna, medan de muntliga tentamina och de s.k. quizzes ansågs vara medelsvåra.
Speciellt intressanta kommentarer	Se bilaga.

Synpunkter från övriga lärare efter avslutad kurs

Vad fungerade bra	-
Vad fungerade mindre bra	-

Resultat av kursnämndsmöte efter examination

Studenternas sammanfattn.	-
Förslag till förändringar	-
Länk till kursnämndsprot.	-

Kursansvarigs sammanfattande berättelse

Helhetsintryck	Jag är i stort sett mycket nöjd med utfallet av kursen. Studenterna hade, som tidigare år, liknande förkunskaper. Antalet studenter var något fler än jämfört med föregående år.
Positiva synpunkter	Se bilaga med resultat av kursenkät.
Negativa synpunkter	Se bilaga med resultat av kursenkät.
Syn på förkunskaperna	-
Syn på undervisningsformen	-
Syn på kurslitt/kursmaterial	Läroboken, som har använts tolv gånger tidigare, fungerar bra att använda som kurslitteratur i kursen, vilken är Tommy Ohlsson, Relativistic Quantum Physics (Cambridge University Press, 2011).
Syn på examinationen	Jag är på det stora hela nöjd med hur examinationen har fungerat och har inga större planer på att förändra den tills nästa kursomgång.

Kursens pedagogiska utveckling II Obligatorisk del

Hur förändringarna till denna kursomgång fungerade	-

Förändringar som bör göras inför nästa kursomgång	Jag tycker att det vore bättre att kursen skulle ges under två perioder istället för en period, eftersom det ger studenterna mer tid att "smälta" kursmaterialet som uppfattas som svårt i allmänhet av studenterna.

Övrigt

Kommentarer

Bilagor:

1. Kurs-PM: SI2390 Relativistic Quantum Physics, 7.5 ECTS credits
2. Obligatoriska inlämningsuppgifter [Homework Problems #1-#3 in SI2390 Relativistic Quantum Physics, 7.5 credits – Spring 2024, Periods 3]
3. Resultat av: Course evaluation

Instruktioner till kursanalysformulär

1) Kursanalysformuläret fylls i interaktivt; fälten expanderar automatiskt.
2) Fyll i fälten inom en månad efter kursens slut. (Viktigt krav från KTH!)
   Skicka sedan till studierektor (som vidarebefordrar till prefekt och programansvarig).
3) Försök att ge så kompletta uppgifter som möjligt.
   Tänk på att kursanalysen är ett hjälpmedel inte bara för teknologerna, utan även för Dig som lärare.
4) Med "prestationsgrad" avses antalet presterade poäng hittills på kursen (inlämningsuppgifter, projektuppgifter, laborationer etc.) dividerat med antalet möjliga poäng för de registrerade studenterna.
   Med "examinationsgrad" avses antalet studenter av de registrerade, som klarat samtliga kurskrav.
   Kurssekreteraren hjälper gärna till här.
5) Kontakten med studenterna:
- Etablera kursnämnd under kursens första vecka (minst två studerande, gärna genusbalanserad).
- Lämplig bonus till kursnämndsdeltagarna är fri kurslitteratur.
- Om kursnämnd ej kan etableras, skall sektionens studienämndsordförande (SNO) kontaktas genast (se www.ths.kth.se/utbildning/utbildningsradet.html för kontaktuppgifter).
- Kursnämnden skall sammanträda under kursens gång, exempelvis i halvtid. Har mittkursutvärdering genomförts, skall den diskuteras då.
- Kursnämnden skall även ha ett möte efter det att studenterna har besvarat kursutvärderingen och kursnämndens studenter fått tillgång till resultaten. Undantaget är kurser i period fyra, där mötet bör ske direkt efter examinatioinen är avslutad för att analysen skall vara klar innan sommaren.
- Under det avslutande kursnämndsmötet bör studenterna föra protokoll. Detta protokoll skall kursansvarig få senast en vecka efter mötet.
- Det är kursansvarigs ansvar att kalla till kursnämndsmöten.

Slutligen, tänk på:
- det är viktigt att kursanalysen tydligt *visar utvecklingen av kursens kvalitet* från ett läsår till nästa.
- möjligheten att lägga ut kursanalysen på kurshemsidan.
- spara kursanalysen till förberedelsearbetet inför nästa kursomgång.

Department of Physics

SI2390 Relativistic Quantum Physics
7.5 credits
Spring 2024, Period 3

General Information

"Relativistic Quantum Physics" is a course where important theories for elementary particle physics and symmetries are learned. During the course, it will be illustrated how relativistic symmetries and gauge symmetries can restrict "possible" theories. The course will give an introduction to perturbation theory and Feynman diagrams. The problem with divergencies will be mentioned and the concepts for regularization and renormalization will be illustrated.

Intended Learning Outcomes

After completion of the course you should be able to:

- apply the Poincaré group as well as classify particle representations.
- analyze the Klein–Gordon and the Dirac equations.
- solve the Weyl equation.
- know Maxwell's equations and classical Yang–Mills theory.
- quantize Klein–Gordon, Dirac, and Majorana fields as well as formulate the Lagrangian for these fields.
- use perturbation theory in simple quantum field theories.
- formulate the Lagrangian for quantum electrodynamics as well as analyze this.
- derive Feynman rules from simple quantum field theories as well as interpret Feynman diagrams.
- analyze elementary processes in quantum electrodynamics.
- compute radiative corrections to elementary processes in quantum electrodynamics.

Syllabus

The course consists of two parts:

Part I. Relativistic quantum mechanics: Tensor notation. The Lorentz and Poincaré groups. Casimir operators. Irreducible representations of particles. The Klein–Gordon equation. The Dirac equation. The structure of Dirac particles. The Dirac equation: central potentials. The Weyl equation.

Part II. Introduction to relativistic quantum field theory: Neutral and charged Klein–Gordon fields. The Dirac field. The Majorana field. Maxwell's equations and quantization of the electromagnetic field. Introduction to Yang–Mills theory. Asymptotic fields: LSZ formulation. Perturbation theory. Introduction to quantum electrodynamics. Interacting fields and Feynman diagrams. Elementary processes of quantum electrodynamics. Introduction to regularization, renormalization, and radiative corrections.

Prerequisites

The following courses are mandatory:

- Quantum Physics
- Special Relativity Theory

The following course is recommended:

- Classical Theoretical Physics

Lectures and Lecturers

The course contains 36 h lectures (18×2 h), which will be given in English. The lecturers and course responsibles of the course are:

Professor Sandhya Choubey (lectures 1–12)
Department of Physics, KTH Royal Institute of Technology
Visiting address: Roslagstullsbacken 21, floor 5, room A5:1025
E-mail: `choubey@kth.se`

Professor Tommy Ohlsson (lectures 13–18)
Department of Physics, KTH Royal Institute of Technology
Visiting address: Roslagstullsbacken 21, floor 5, room A5:1029
Telephone: 08-7908261
E-mail: `tohlsson@kth.se`

The examiner of the course is: **Professor Tommy Ohlsson**

Course Literature

The course literature consists of one book (mainly):

- T. Ohlsson, *Relativistic Quantum Physics – From Advanced Quantum Mechanics to Introductory Quantum Field Theory*, Cambridge (2011)

Further recommended reading:

- A.Z. Capri, *Relativistic Quantum Mechanics and Introduction to Quantum Field Theory*, World Scientific (2002)
- W. Greiner, *Relativistic Quantum Mechanics – Wave Equations*, Springer (2000)
- F. Gross, *Relativistic Quantum Mechanics and Field Theory*, Wiley (1993)
- F. Mandl and G. Shaw, *Quantum Field Theory*, rev. ed., Wiley (1994)
- J. Mickelsson, T. Ohlsson, and H. Snellman, *Relativity Theory*, KTH (2005)
- M.E. Peskin and D.V. Schroeder, *Introduction to Quantum Field Theory*, Harper-Collins (1995)
- F. Schwabl, *Advanced Quantum Mechanics*, Springer (1999)
- S.S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, Dover (2005)
- F.J. Ynduráin, *Relativistic Quantum Mechanics and Introduction to Field Theory*, Springer (1996)

Requirements

Hand in assignments (INL1; 4.5 ECTS credits) and an oral exam (TEN1; 3 ECTS credits).

Examination

The examination of the course will be a combination of homework problems (INL1) and an oral examination (TEN1).

INL1. There will be three sets of homework problems, which each consists of two parts, during the course. These will be distributed and should be handed in according to the following scheme:

Homework Problems	Out	In
Set # 1 – Part I	Lecture 5 (January 24, 2024)	January 31, 2024 @ 14:00–15:00
Set # 1 – Part II	Lecture 5 (January 24, 2024)	February 7, 2024 @ 23:59
Set # 2 – Part I	Lecture 11 (February 6, 2024)	February 14, 2024 @ 14:00–15:00
Set # 2 – Part II	Lecture 11 (February 6, 2024)	February 20, 2024 @ 23:59
Set # 3 – Part I	Lecture 17 (February 27, 2024)	March 5, 2024 @ 14:00–15:00
Set # 3 – Part II	Lecture 17 (February 27, 2024)	March 12, 2024 @ 23:59

TEN1. The oral examinations will take place after the last lecture of the course. Each examination will take approximately 15 minutes. The time for the examination will be agreed upon between the student and the examiner.

Grading

The different grades are: A, B, C, D, E, Fx, and F. The grades will be awarded according to the following scheme:

Grade	Homework Problems (INL1)	Oral Examination (TEN1)
F	$<$ 50 % of all problems correct	Failed
Fx	$<$ 50 % of all problems correct	Passed
Fx	$\geq$ 50 % of all problems correct	Failed
E	$\geq$ 50 % of all problems correct	Passed
D	$\geq$ 60 % of all problems correct	Passed
C	$\geq$ 70 % of all problems correct	Passed
B	$\geq$ 80 % of all problems correct	Passed
A	$\geq$ 90 % of all problems correct	Passed

In addition, you need to obtain at least 50 % on each homework problem set in order to obtain a passing grade (E or higher). If you obtain a total result of more than 50 %, but do not fulfill this criterion, you will be given the grade Fx and a chance to make a completing task for the grade E.

Good luck with the course!

Schedule and Program of Lectures

Schedule of Lectures

#	Week	Day	Date	Time	Room	Contents
1	3	Tuesday	January 16, 2024	10-12	FD41	Introduction
2		Wednesday	January 17, 2024	15-17	FD41	General description of relativistic states
3		Friday	January 19, 2024	10-12	FD41	The Klein–Gordon equation
4	4	Monday	January 22, 2024	10-12	FD41	The Dirac equation
5		Wednesday	January 24, 2024	15-17	FD41	—”—
6		Friday	January 26, 2024	10-12	FD41	The Dirac equation
7	5	Monday	January 29, 2024	10-12	FD41	Quantization of the non-relativistic string
8		Tuesday	January 30, 2024	10-12	FD41	Introduction to relativistic quantum field theory
9		Friday	February 2, 2024	10-12	FD41	Quantization of the Klein–Gordon field
10	6	Monday	February 5, 2024	10-12	FD41	Quantization of the Dirac field
11		Tuesday	February 6, 2024	10-12	FD41	Maxwell’s equations and quantization of the electromagnetic field
12		Friday	February 9, 2024	10-12	FD41	Introduction to Yang–Mills theory
13	7	Tuesday	February 13, 2024	10-12	FD41	Asymptotic field and the LSZ formalism
14		Friday	February 16, 2024	10-12	FD41	Perturbation theory
15	8	Tuesday	February 20, 2024	10-12	FD41	—”—
16		Friday	February 23, 2024	10-12	FD41	—”—
17	9	Tuesday	February 27, 2024	10-12	FD41	Elementary processes of quantum electrodynamics
18		Friday	March 1, 2024	10-12	FB51	Introduction to regularization, renormalization, and radiative corrections

Program of Lectures

Below, RQP refers to the textbook T. Ohlsson, *Relativistic Quantum Physics – From Advanced Quantum Mechanics to Introductory Quantum Field Theory*, Cambridge (2011).

Lecture 0: Refresh chapter 1 in Mickelsson, Ohlsson & Snellman or some similar material on special relativity theory!

Lecture 1: Tensor notation. The Lorentz and Poincaré groups. Casimir operators. Irreducible representations of particles.
Literature: Chapter 1 in RQP.

Lecture 2: General description of relativistic one-particle states.
Literature: Chapter 1 in RQP.
For the interested student: E. Wigner, *On Unitary Representations of the Inhomogeneous Lorentz Group*, Ann. Math. **40**, 149 (1939).

Lecture 3: The Klein–Gordon equation. The Klein paradox.
Literature: Chapter 2 in RQP.

Lecture 4: The Dirac equation. Gamma "gymnastics".
Literature: Chapter 3 in RQP.

Lecture 5: The structure of Dirac particles. The Dirac equation: central potentials.
Literature: Chapter 3 in RQP.

Lecture 6: The hydrogenic atom. The Weyl equation.
Literature: Chapter 3 in RQP.

Lecture 7: Quantization of the non-relativistic string.
Literature: Chapter 4 in RQP.

Lecture 8: Introduction to relativistic quantum field theory.
Literature: Chapter 5 in RQP.

Lecture 9: Neutral and charged Klein–Gordon fields.
Literature: Chapter 6 in RQP.

Lecture 10: The Dirac field. The Majorana field.
Literature: Chapter 7 in RQP.

Lecture 11: Maxwell's equations and quantization of the electromagnetic field.
Literature: Chapter 8 in RQP.

Lecture 12: Introduction to Yang–Mills theory.
Literature: Chapter 9 in RQP.

Lecture 13: Asymptotic fields: LSZ (Lehmann–Symanzik–Zimmermann) formulation.
Literature: Chapter 10 in RQP.

Lecture 14: Perturbation theory.
Literature: Chapter 11 in RQP.

Lecture 15: see lecture 14.
Literature: Chapter 11 in RQP.

Lecture 16: Introduction to quantum electrodynamics. Interacting fields and Feynman diagrams.
Literature: Chapter 11 in RQP.

Lecture 17: Elementary processes of quantum electrodynamics.
Literature: Chapter 12 in RQP.

Lecture 18: Introduction to regularization, renormalization, and radiative corrections.
Literature: Chapter 13 in RQP.

Homework Problems #1
SI2390 Relativistic Quantum Physics, 7.5 credits
Spring 2024, Period 3

Deadlines: Part I: Quiz on January 31, 2024 @ $14:00-15:00$
Part II: February 7, 2024 @ $23:59$

Examiners: Prof. Sandhya Choubey(choubey@kth.se)
Prof. Tommy Ohlsson(tohlsson@kth.se)

Part I

1. (a) Calculate: $Tr(\gamma^\mu\gamma^\nu)$, $Tr(\gamma^\mu\gamma^\nu\gamma^\rho)$, $Tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\lambda)$, $Tr(\gamma^\mu\gamma^5)$, $Tr(\gamma^\mu\gamma^\nu\gamma^5)$
   (b) Calculate: $\gamma^\mu\gamma^\nu\gamma_\mu$, $\not{p}\gamma^\mu\not{p}$, $\gamma^\mu\gamma^5\gamma_\mu\gamma_5$, $\gamma^\mu\gamma^\alpha\gamma^\beta\gamma_\mu$
2. (a) Find the Hamiltonian of a free Dirac particle whose equation is given as $(i\gamma^\mu\partial_\mu - m\mathbb{1}_4)\psi(x) = 0)$.
   (b) Find the commutation relation of this Hamiltonian with the operators $\mathbf{L} = \mathbf{x}\times\mathbf{p}$, $\mathbf{S} = \boldsymbol{\Sigma}/2$, where $\boldsymbol{\Sigma} = i\gamma\times\gamma/2$ and $\mathbf{J} = \mathbf{L}+\mathbf{S}$.

Part II

1. Consider a theory with two interacting real scalar fields ϕ_1 and ϕ_2. Let this theory be described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m_1^2\phi_1^2 - \frac{1}{2}m_2^2\phi_2^2 - \lambda\phi_1^2\phi_2^2. \quad (1)$$

   (a) What are the symmetries of this Lagrangian? (0.5 point)
   (b) Find the corresponding equations of motion, starting from the principle of least action. (1 point)
2. The chirality operator for a Dirac fermion is defined as γ^5 while the helicity operator is defined as $\mathbf{s}\cdot\mathbf{p}$, where $\mathbf{s}$ is the particle spin and $\mathbf{p}$ the direction of it's motion.
   (a) Which of these operators is/are Lorentz invariant? Explain. (0.5 point)
   (b) Which of these operators is/are a conserved quantity? Explain. (0.5 point)
   (c) Under what condition are they essentially the same? Discuss. (0.5 point)
3. Calculate $Tr[(\not{q}+m)\gamma^\mu(1-\gamma^5)(\not{p}+m)\gamma^\nu(1-\gamma^5)]$ in terms of the 4-momenta p and q. (1 point)

Rules and guidelines for homework problems

When solving the homework problems, you are allowed to use books or other sources of information, as well as to discuss the problems with one another. However, the solutions that you hand in have to reflect your own knowledge. Therefore, make sure that you motivate the steps in your solutions. If we receive nearly identical solutions, we might ask you to orally describe what you have done.

Also, you should follow the following simple guidelines:

- **Motivate your computations, it should be clear that you understand what you are doing and why!**
- Start each problem on a separate sheet of paper
- Write clearly, so that your solutions are readable
- Do not forget to put your name on your solutions

Failure to follow these rules will result in a deduction of points.

Homework Problems #2
SI2390 Relativistic Quantum Physics, 7.5 credits
Spring 2024, Period 3

Deadlines: Part I: Quiz on February 14, 2024 @ $14:00-15:00$
Part II: February 20, 2024 @ $23:59$
Examiners: Prof. Sandhya Choubey(choubey@kth.se)
Prof. Tommy Ohlsson(tohlsson@kth.se)

Part I

1. a) Suppose the field $\phi = \phi(t, z)$, where t is time and z describes a spatial direction, is a solution to the two-dimensional Klein-Gordon equation $(\Box_2 + m^2)\phi(t, z) = 0$ (where $\Box_2 \equiv \partial_0^2 - \partial_z^2$), with periodic boundary conditions and expanded in normal modes as

$$\phi(t, z) = \sum_{n=-\infty}^{\infty} c_n[a_n\phi_n(t, z) + a_n^\dagger \phi_n^*(t, z)], \quad \pi(t, z) = \dot{\phi}(t, z), \tag{1}$$

where the operators a_n and $a_n^\dagger$ satisfy the commutation relations

$$[a_n, a_{n'}] = [a_n^\dagger, a_{n'}^\dagger] = 0, \quad [a_n, a_{n'}^\dagger] = \delta_{nn'} \tag{2}$$

the states $\phi_n(t, z) = \frac{1}{\sqrt{l}} e^{i(k_n z - \omega_n t)}$ ($k_n = 2\pi n/l,\ n = 0, \pm 1, \pm 2, ...$ and $\omega_n > 0$) and

$$\int_0^l \phi_n^*(t, z)\phi_m(t, z)dz = \delta_{nm}. \tag{3}$$

Find the co-efficients c_n, which will ensure that the canonical commutation relations assume the standard form

$$[\phi(t, z), \pi(t, z')] = i\delta(z - z') \tag{4}$$

Are any other assumptions needed?

b) Find the equation of motion for the real scalar field $\phi = \phi(x)$, where $x = x^\mu$ is the 4-position vector, using the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 \tag{5}$$

If the field $\phi(x)$ is reduced to a two-dimensional one $\phi(t, z)$, how will the equation of motion change and how does it compare to the one in Problem a)?

2. If the changed field $\phi = (\phi_1 + i\phi_2)\sqrt{2}$, where ϕ_1 and ϕ_2 are commuting hermitian fields, show that the Lagrangian density for the charged field ϕ,

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi, \tag{6}$$

can be written as the sum of two *independent* Lagrangian densities

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 \tag{7}$$

What is the form of each density $\mathcal{L}_i$?

Part II

1. The Lagrangian for electrons is given by $\mathcal{L}_0 = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$ ($D_\mu = \partial_\mu + ieA_\mu$) is invariant under U(1) gauge transformations.
   (a) Find (derive) the corresponding Noether current.
   (b) What is the corresponding conserved charge?

   (1 point)

2. In the Lagrangian for a complex scalar field, replace the partial derivative ∂_μ with $D_\mu = \partial_\mu + ieA_\mu$.
   (a) Show that the resultant Lagrangian is invariant under gauge transformations.
   (b) Find the interaction terms of the scalar field with A_μ.

   (1 point)

3. The Yang-Mill's Lagrangian is given by

$$\mathcal{L}_{YM} = -\frac{1}{4} F^{a\,\mu\nu} F^a_{\mu\nu}, \tag{8}$$

   where $F^a_{\mu\nu}$ is the non-Abelian gauge field strength tensor corresponding to the gauge group SU(2).
   (a) Show that this Lagrangian is gauge invariant.
   (b) Expand the Lagrangian completely in terms of the gauge field A^a_μ.
   (c) What is the physical interpretation of each of the terms in the Lagrangian.
   (d) How does the Lagrangian change if we consider a U(1) gauge symmetry instead of SU(2)?

   (2 points)

Rules and guidelines for homework problems

When solving the homework problems, you are allowed to use books or other sources of information, as well as to discuss the problems with one another. However, the solutions that you hand in have to reflect your own knowledge. Therefore, make sure that you motivate the steps in your solutions. If we receive nearly identical solutions, we might ask you to orally describe what you have done.

Also, you should follow the following simple guidelines:

- **Motivate your computations, it should be clear that you understand what you are doing and why!**
- Start each problem on a separate sheet of paper
- Write clearly, so that your solutions are readable
- Do not forget to put your name on your solutions

Failure to follow these rules will result in a deduction of points.

Department of Physics

Homework Problems #3 in SI2390 Relativistic Quantum Physics, 7.5 credits Spring 2024, Period 3

Deadline: Part I: Quiz on March 5, 2024 @ 14:00–15:00
Part II: March 12, 2024 @ 23:59

Examiner: Prof. Tommy Ohlsson
Telephone: 08-790 8261 • E-mail: `tohlsson@kth.se`

GOOD LUCK!

Part I

1. a) For a free Klein–Gordon field ϕ, compute the following correlation functions

$$\langle 0|T[\phi(x)\phi(y)\phi(z)]|0\rangle \quad \text{and} \quad \langle 0|T[\phi(x)\phi(y)\phi(z)^4]|0\rangle,$$

where $|0\rangle$ is the ground state (vacuum) of the free Klein–Gordon theory.

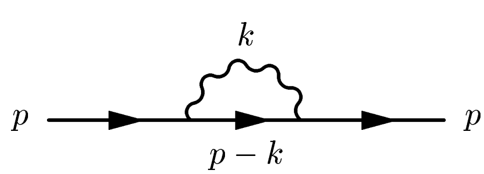
Note that the correlation function $\langle 0|T[\phi(x)\phi(y)\phi(z)^4]|0\rangle$ would only appear within an interacting theory, but this correlation function can still be computed.

b) For a free Dirac field ψ, compute the following correlation functions

$$\langle 0|T[\psi(x)\psi(y)\psi(z)]|0\rangle \quad \text{and} \quad \langle 0|T[\psi(x)\psi(y)\bar{\psi}(z)\bar{\psi}(w)]|0\rangle,$$

where $|0\rangle$ is the ground state (vacuum) of the free Dirac theory.

2. *Electron self-energy.* Write down the amplitude for the Feynman electron self-energy diagram shown in the following figure:



Part II

1. *Decay of a scalar particle.* Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 - \mu\Phi\phi^2.$$

   The last term is an interaction that allows a Φ particle to decay into two ϕ particles, provided that $M > 2m$. Assuming that this condition is fulfilled, calculate the lifetime of the Φ particle to lowest order in the coupling constant μ.

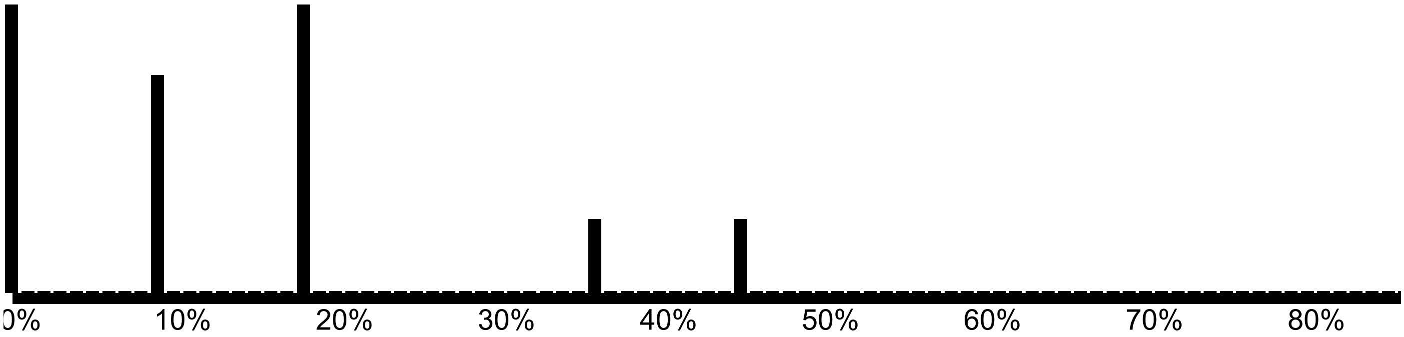
2. *Pseudoscalar Yukawa theory.* One part of the interaction Lagrangian for pseudoscalar Yukawa theory is given by

$$\mathcal{L}_{PY,\,\mathrm{int.}} = -\mathrm{i}g\bar{\psi}\gamma^5\psi\phi,$$

   where ϕ is a neutral Klein–Gordon field, ψ is a Dirac field, and g is the coupling constant of the interaction. Using dimensional regularization, determine the one-loop vertex correction to this interaction. In addition, calculate the counterterm δg.

Quiz Summary

Average Score	High Score	Low Score	Standard Deviation	Average Time
0%	0%	0%	1.5	



Question Breakdown

Attempts: 12 out of 13

What is your overall impression of the course?

Very positive	2 respondents	15 %	✓
Quite postive	7 respondents	54 %	
Neutral - no opinion	1 respondent	8 %	
Quite negative	2 respondents	15 %	
Very negative		0 %	
No Answer	1 respondent	8 %	

Attempts: 12 out of 13

How would you rate the difficulty of the course?

Very difficult	4 respondents	31%	✓
Quite difficult	7 respondents	54 %	
Average	1 respondent	8 %	
Easy		0 %	
Very easy		0 %	
No Answer	1 respondent	8 %	

Attempts: 12 out of 13

Has there been much overlap with other courses?

Far too much overlap		0 % ✓
Some overlap, but it was useful to go over the topics again	5 respondents	38 %
Mostly unnecessary overlap	1 respondent	8 %
No overlap	6 respondents	46 %
No Answer	1 respondent	8 %

Attempts: 12 out of 13

How were the quizzes?

Very difficult		0 % ✓
Difficult	5 respondents	38 %
Average	7 respondents	54 %
Easy		0 %
Very easy		0 %
No Answer	1 respondent	8 %

Attempts: 12 out of 13

How were the (hand-in) homework problems?

Very difficult	4 respondents	31% ✓
Difficult	7 respondents	54 %
Average	1 respondent	8 %
Easy		0 %
Very easy		0 %
No Answer	1 respondent	8 %

Attempts: 12 out of 13

How was the oral examination?

Very difficult		0 % ✓
Difficult	3 respondents	23 %
Average	7 respondents	54 %
Easy	1 respondent	8 %
Very easy	1 respondent	8 %
No Answer	1 respondent	8 %

Attempts: 12 out of 13

What is your opinion about the course description and the administration of the course?

Very good	4 respondents	31%	✓
Good	6 respondents	46 %	
Average	1 respondent	8 %	
Poor	1 respondent	8 %	
Very poor		0 %	
No Answer	1 respondent	8 %	

Attempts: 12 out of 13

What is your opinion about the course literature?

Very good	3 respondents	23%	✓
Good	5 respondents	38 %	
Average	3 respondents	23 %	
Poor		0 %	
Very poor	1 respondent	8 %	
No Answer	1 respondent	8 %	

Attempts: 11 out of 13

How were the lectures? (Sandhya Choubey)

Very good	2 respondents	15%	✓
Good	2 respondents	15 %	
Average	5 respondents	38 %	
Poor	2 respondents	15 %	
Very poor		0 %	
No Answer	2 respondents	15 %	

Attempts: 11 out of 13

How were the lectures? (Tommy Ohlsson)

Very good	1 respondent	8%	✓
Good	8 respondents	62 %	
Average	2 respondents	15 %	
Poor		0 %	
Very poor		0 %	
No Answer	2 respondents	15 %	

Attempts: 8 out of 13

Please enter any further comments on the course below.

Ungraded answers 13 respondents 100 %

Report has never been generated.

Report can not be generated for Survey Quizzes.

The quiz/assignment type of examination must be changed. It is unfair that if you miss one quiz/assignment for whatever reason you can only get an E (especially for the quizzes that are given only once). The grade should represent our preparation and in this case I hope you can acknowledge that it doesn't. A student extremely prepared that for a personal reason cannot take a quiz in a cartain day at a certain time will have the same grade as a student with a very weak praparation. I honestly suggest a big oral test (45/60 minutes each) that gives tha total grade. I think in this way every student will have the grade that their preparation deserves. Or at least just keep the assignment and eliminate the quizzes.

The course was interesting but challenging, most concepts were new and it was a pretty big leap from the advanced quantum mechanics/special relativity courses.
The quizzes were fair. Some questions in quiz 3 were tricky as there were some terms (begrepp) which were hard to find a definition for.
The hand-ins were required a lot of time (not in a bad way) and gave a pretty good understanding of the topics they touched on. The 2nd question on hand-in 3 was more challenging than the others. The most difficult part of the hand-ins were to decide how to begin, after that they were mostly straight-forward. The practice (part I) questions were much easier than part II (as they should).
The oral exam was not hard to pass, but was kind of stressful. I think the examinators kept this in mind during grading, as the performance level dropped significantly under pressure (at least for me).
Something that could make the course easier is more short practice exercises to get the general idea for e.g., renormalization, Feynman diagrams, time- and normal-ordering, etc. (just focusing on simple problems).
The course literature was good, renormalization was somewhat difficult to grasp from it but other than that it did a good job of explaining concepts.

Would be good to have some more clarity regarding how the oral exam is done before the exam.
The last homework was very hard since information and explanations around some key information was not metioned at all in the book.

I thought that both Sandhya and Tommy had excellent lectures! Only reason I checked "Good" and not "Very good" on Tommy is because I can find it quite difficult to follow when there is a presentation, and when most things aren't being derived on the board, which Sandhya did most of the time. Otherwise really good impressio of both!:)

The topic of the course was very interesting! The reason for the poor overall impression of the course is due to how the course was administered and evaluated. It would have helped a lot to have some exercises (not only for the quizzes) to use as general practice of the topics we were going through and it would have also helped a lot to have the lecture notes posted on canvas. It was very difficult to practice for the oral exam as very little information was given about how much knowledge was needed and what areas were more or less relevant in the course. Overall, this made it very difficult to navigate the course material.

The last homework problem was very difficult and time consuming. This meant that I had very little time to prepare for the oral exam.

It would be more helpful if the course ran at a 25% pace but in two periods because many concepts required thinking, again and again, to grasp the idea properly. I think if the contents were to be learned in double the time, it would be much more comprehensible.

First of all, I found the content of the course very interesting and especially during the end of the course when I hade time to go through it all I feel like I learned a lot. I must say that my interest in this field has increased, so that is very positive! :) I should also add that this is one of the most challenging courses I have done in my four years at KTH.
This is why I would have enjoyed more encouragement to questions and discussions especially during Sandhya's lectures. I felt like many concepts very new and quite difficult for most of us, but I did not feel that questions were met nicely and I know many people did not really "dare" to ask questions. I would have enjoyed a little prompt in the beginning of the course of something along the lines of "in this course we encourage questions and discussion", I think this really sets the tone and makes a huge difference. Maybe this is something you could think about doing for the next year's course.
Furthermore I did not like the concept of the quizzes. I feel like it was mostly stress about not being able to see all questions and not being able to go back that led me to unnecessary wrong answers, and it does not feel very justified that my grade will be affected by accidentally writing "3" instead of "-3" or forgetting at one point to not put a dagger on an operator even though I calculated as if it was there. The concept of having a one hour examination without having an overview of the questions might suit some people but when talking to my course peers, everyone agreed on this being a bad way of examining people. I also happen to know that this is a complaint that people the past years have expressed, which makes me a bit curious as to how you actually treat these course evaluations and student influence on education....
With this said, I liked the homework assignments, I feel like they were on a good (quite difficult) level but having a week or so to do them felt fair and a good ground for examination. So maybe for next year it would be an improvement to keep the quizzes but include a possibility of having an overview of the questions, I think this would reduce stress and result in better answers from most students.